Solution for the Mathematical Model for the Control of Lassa fever using the Revised Adomian Decomposition Method

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Abstract
Adjacent to the terminal transmissible sickness recognized as Ebola fever, there is another one called the Lassa fever. This disease kills move pregnant women as Ebola does. We employed the Revised Adomian decomposition method to solve and analyze the control of transmission of the SIR model of this disease. The relationship among the susceptible group, infectious group and the recovered group were analyzed under 3 cases which consists four sets of parameters; the birth rate B, the natural death rate D, the transmission rateα and the recovered rate Υ. The disease control as used here is based on the reproductive number. One of the variables that kick-start the solution of the model, which is less than one means the disease is eradicated.

Keywords: *System of Volterra integral equations, Adomian polynomials; Revised Adomian decomposition; Basic reproductive number.*

1. Introduction

Mathematical modeling has become very important tools in analyzing the spread and control of infectious diseases. According to Laarabi et al. (2013), Mathematical modeling is of considerable value of the under lying mechanism which influences the spread of the diseases and suggest control strategies. Bernoulli and Blower (2000) states that mathematical models provide conceptual results such as thresholds, basic reproductive numbers, contact numbers and replacement numbers. Modeling the transmission of infectious diseases provides a means of assess the effectiveness of control interventions and develop more efficient means to stop the spread of the disease in the future.

Lassa fever is a viral hemorrhagic fever that is often spread by multimamate rats. It is an agent that causes infection to human with high death rates. Despite all efforts put in place by various government and health agencies to control the disease, the virus continues to spread.

A lot of problems in Physics, Biology and Engineering are modelled by system of differential equations which are solved by semi-analytical methods like Adomian decomposition method (Biazar et al., 2012), Homotopy perturbation method (Biazar & Ghanberi, 2012), New

iterative method(Aboiyar & Ibrahim, 2015), differential transform method (Keskin & Oturan,2010), and Taylor allocation method, among others. Among the above mentioned methods, the Revised Adomian decomposition method (RAMD) is very simple in its principles and application to solve system of non-linear differential equations as it does not generate secular terms or rely on trial functions as others does.

In this paper, we apply the RADM to investigate the solution of the control of a mathematical model of Lassa fever proposed by Ogebi et al. (2012), considering different reproductive numbersR_0.
The paper has been organized as follows; in section 2, we give the basic definitions of the basic reproductive number R. In section 3, we stated the SIR model of Lassa fever as proposed by Ogebi et.al. Section 4 deals with the analysis of RAMD applied to a system of ordinary differential equations. In section 5, we presented the numerical results of the model with respect to various reproductive numbers using RAMD. This is followed by the discussions of the results in section 6. Lastly, the conclusion was presented in section 7.

2. The Basic Reproductive Number

The basic reproductive number R_0 is the parameter that determines if the disease will be eradicated or not, this serves as the control strategy of the mathematical model. For the SIR model with vital dynamics, R_0 is defined as the product of birth rate B with transmission rate α and the duration of infectious period $\frac{1}{D+\Upsilon}$ and divide by death rate D.

$$R_0 = \frac{B\alpha}{D(D+\Upsilon)}$$

3. Sir Model of Lassa fever

The SIR model can be represented by a system of ordinary differential equation [10] as follows;

$$\begin{aligned} \frac{dy}{dt} &= -\alpha SI + B - DS \\ \frac{dI}{dt} &= \alpha SI - (\Upsilon + D)I \\ \frac{dR}{dt} &= \Upsilon I - DR \end{aligned} \qquad (1)$$

S (0) = 0.5000, I (0) = 0.3226, R (0) = 0.1774, where S, I and R represent the susceptible group, infectious group and the recovered group respectively. B is the birth rate; D is the death rate, α is the transmission rate of the disease and Υ is the recovery rate of the infectious group after vaccination.
The system (1) is equivalent to the system of Voltera integral equation

$$\begin{aligned} S(t) &= S(0) + \int_0^t (-\propto SI + B - DS)dt \\ I(t) &= I(0) + \int_0^t (\propto SI - (\Upsilon + D)I)dt \ldots (2) \\ R(t) &= R(0) + \int_0^t (\Upsilon I - DR)dt \\ S(0) &= 0.5000, I(0) = 0.3226, R(0) = 0.1774. \end{aligned}$$

4. RADM for a System of Ode

In this section, the modification of the Adomian decomposition method [], was presented.
They set

$$U_{10}(x) = c_1 + \int_0^1 g_1(x)dx$$

$$U_{1,m+1}(x) = \int_0^x \sum_{j+1}^{n} \mathrm{b_{1j}}\,(\mathrm{x})\mathrm{U_{jm}}\mathrm{dx} + \int_0^x \mathrm{A_{1m}}dx$$

$$U_{10}(x) = c_1 + \int_0^1 g_1(x)dx \int_0^x \sum_{j+1}^{l-1} \mathrm{b_{lj}}\,(\mathrm{x})\mathrm{U_{j0}}\mathrm{dx}, \mathrm{l} = 2,3,\ldots$$

$$U_{1,m+1}(x) = \int_0^1 \sum_{j+1}^{l-1} b_{lj}\,(x)U_{j,m+1}dx + \int_0^x \sum_{j+1}^{n} b_{1j}U_{jm}dx + \int_0^x A^*_{lm}dx$$

where A^*_{im}, I is defined as

$$A^* = \begin{cases} A_{im+1} \quad if\ u_l\ are\ independent\ of\ u_l, u_{l+1,} \ldots, u_n \\ 1_{A_{l,m+1}} + 2_{A_{l,m}}\ if\ u_l\,(u_1, \ldots, u_n) = 1_{N_l}(u_1, \ldots, u_{l-1}) + 2_{N_2}(u_1, \ldots, u_n) \quad l = 2,3,\ldots \\ A_{lm}\ otherwise \end{cases}$$

Here, $1_{A^*_{A_{l,m+1,}}} 2_{A_{l,m}}$ *are Adomian polynomials corresponding to* 1_{N_l} *and* 2_{N_2}.

5. Numerical Simulation

To execute this task, the parameter B, D, α, and γ defined in equation (1) are assigned values from hypothetical data [10]. The birth rate and death rate will be of same rates and of different rates in some cases. The total population,$u = S + R + I = 1$, where $S(0) = 0.5000$, $\mathrm{I}\,(0) = 0.3226$,$\mathrm{R}\,(0) = 0.1774$.

The Revised Adomian scheme for equation (2) would lead to,

$$S_0(t) = 0.5 + \int_0^t Bdt; S_{m+1}(t) = \int_0^t (-\alpha S_m I_m - DS_m)dt$$

$$I_0(t) = 0.3226; I_{m+1}(t) = \int_0^t (\alpha S_m I_m - (\Upsilon + D)I_m)\,dt$$

$$R_0(t) = 0.1774 + \int_0^t \Upsilon I_0 dt \ldots (3)$$

$$R_{m+1}(t) = \int_0^t (\Upsilon_{m+1} - DR_m)dt$$

The numerical experiments studied the model in equation (3) under the following cases;

Case 1: B=0.6, D=0.5, ϒ=0.7 and α=0.5. R_0 is calculated to be 0.55<1. The sum of the first four iterations gives:

$\mathrm{S(t)} = 0.5 + 0.4306500000t - 0.8267000000t^2 + 0.01396933942t^3 - \cdots$

$\mathrm{I\,(t)} = 0.3226 + 0.2742100000t + 0.1992055000t^2 - 0.06530243942t^3 + \cdots$

$\mathrm{R\,(t)} = 0.1774 + 0.1371200000t - 0.1302535000t^2 + 0.06819020000t^3 - \cdots$

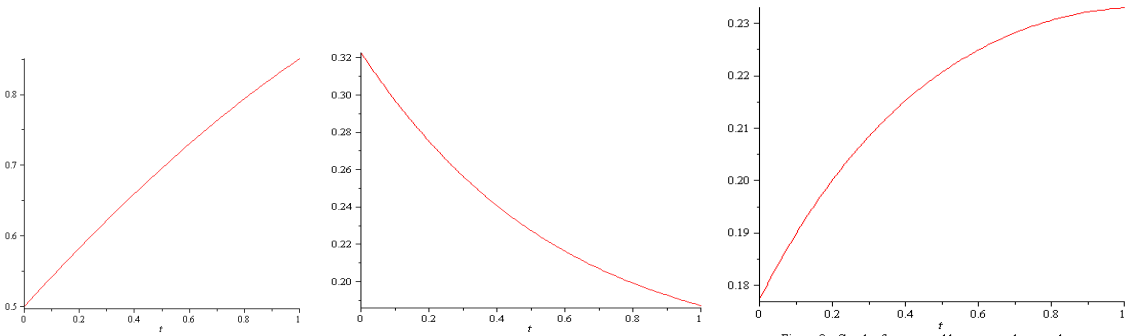

Figure 3a: Graph of susceptible humans under case 1 Figure 3b: Graph of infectious humans under case 1 Figure 3c: Graph of recovered humans under case 1

Case 2: B = 0.5, D = 0.5, and α = 0.5 ,$\gamma = 0.7$. R_0 *is calculated to be* $0.46 < 1$.
We obtain the sum of the first four iterations to be;

$$S(t) = 0.5 + 0.3306500000t - 0.0746050000t^2 + 0.0002310195726t^5 - \cdots$$
$$I(t) = 0.3226 - 0.3064700000t + 0.2242070000t^2 + 0.0050978526256t^4 - \cdots$$
$$R(t) = 0.1774 + 0.1371200000t - 0.141544000t^2 - 0.0213091350t^4 + \cdots$$

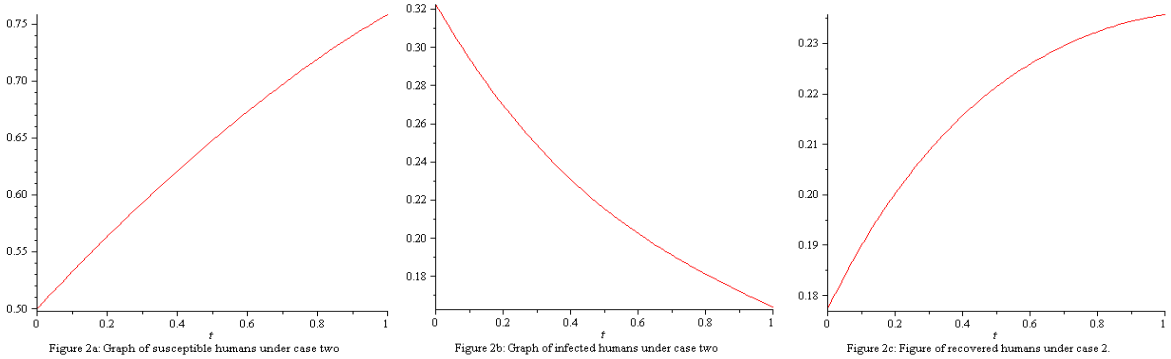

Figure 2a: Graph of susceptible humans under case two Figure 2b: Graph of infected humans under case two Figure 2c: Figure of recovered humans under case 2.

Case 3: B=0.46, D=0.31, α=0.62= and ϒ=0.17. R_0 *is calculated to be* $1.92 > 1$.
The sum of the first four iterations is;

S (t) $=0.5 + 0.3050000000t - 0.04597920000t^2 + 0.0002766721376t^4 + \cdots$
I (t) $=0.3226 - 0.05484200000t + 0.05916484000t^2 + 0.0000305296267t^4 - \cdots$
R (t) $=0.1774 - 0.000015200000t - 0.004638010000t^2 - 0.0006928966788t^4 - \cdots$

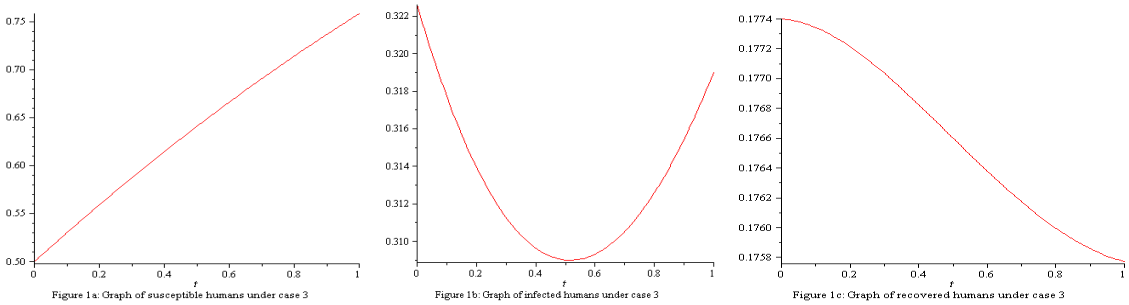

Figure 1a: Graph of susceptible humans under case 3 Figure 1b: Graph of infected humans under case 3 Figure 1c: Graph of recovered humans under case 3

6. Discussion of Results

CASE 1:

Here, we investigate the situation when the birth rates B and the natural death rate are of different rate. Also, the basic reproductive number R_0 is less than one. The clarification of the effect of the solution can be seen in figure 1-3. From figure 1-2 and table 1, the infectious

group population reducing in number as susceptible group population increase in number. At a time the infectious group population reduced to zero at the highest population value of the susceptible group. This means that Lassa fever has been eradicated and its effect can't be noticed anymore. The figure 3 is graphical information of the recovered group. The recovered group population was increasing in number as infectious group decreased in number. At a particular period of time, the trend changed the recovered group in population as the infectious groups reduced to zero. This implies that the vaccine given to Lassa fever patients and control measures put in place to check the spread of the disease were very much effective in controlling the transmission of the Lassa fever virus.

CASE 2:

In this case, we study a situation where the birth rate and the natural death rate are of the same rates. The basic reproductive number in this case is also less than one. From figure 4-6, it can be observed that the infectious group decreasing in population as susceptible group increased, after some months it reduced to zero when susceptible was at the highest value of the population. It was clearly shown; the disease eradicated from the population and the transmission rate was highly controlled to reduce minima. In figure 4-5 and table 2, we observed that the recovered group was decreasing in population, but after some months, the recovered group was reducing in population as infectious group was decreasing in population, and no need to purchase more drugs on this type of disease any longer. At the end, the whole population will be in the susceptible group.

CASE 3:

In this simulation, we study a case where the birth rate is greater than the natural death rate and the transmission rate is four times than the recovering rate. The basic reproductive number is calculated to be R_0=1.9271, figure 7-8 and table 3 illustrates that at the early month, the infectious group was decreasing in population as the susceptible group was increasing in population. The infectious group later begins to increase in population as susceptible group increase in population. There was a change after some months; the susceptible group was decreasing as the infectious group was increasing. At a time both susceptible group and infectious group were constant in their population. Some months later, both recovered group and infectious group were neither increased nor decreased in their population. In figure 7 and 9, four effects were noticed among the recovered group and the susceptible group;

- **(i)** The recovered group was decreasing in population as the susceptible group increases.
- **(ii)** After some months, the recovered group was increasing as the susceptible group was increasing.
- **(iii)** Some months later, the susceptible group was decreasing in population as the recovered group was increasing.
- **(iv)** At later month, both groups neither increase nor decrease in population.

From figure 7, non-linearity relation of three group population is very leash. And the diseases will never be eradicated because the infectious group population nor reduced to zero, due to the basic reproductive number that is higher than one.

7 Conclusion and Recommendation

Conclusion:

In this study, we successfully solved the model of controlling Lassa fever virus that arises from system of ODE using Revised Adomian decomposition method. It was clearly shown

also that to control the spread of Lassa fever in the endemic areas, the basic reproductive number must be below 1, if it is more than 1, the disease will not die out from the area. This is possible if the transmission rate is very low compared to the recovery rate.

Recommendations:

Based on the finding of this research work, we recommend that;

. To effectively prevent and control the fatal reoccurrence of the Lassa fever, the federal government needs to embark on a low cost housing scheme project, to reduce the number of people in a room; this will enhance the reduction of transmission rate.

. To save lives of susceptible humans, good health policy must be implemented, this must start with health personnel, by educating them on how to treat patient with Lassa fever.

. Every Lassa fever patients must be isolated from other patients in hospitals, with proper monitoring and vaccination to increase the rate of recovery and goal of attaining high immunity.

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APPENDIX

List of Table

Table 1: Numerical solution of the model of CASE 1

T	S(t)	I(t)	R(t)
0	0.5000	0.3226	0.1774
0.1000	0.5423	0.2971	0.1899
0.2000	0.5829	0.2752	0.2001
0.3000	0.6220	0.2565	0.2085
0.4000	0.6596	0.2407	0.2153
0.5000	0.6957	0.2274	0.2207
0.6000	0.7302	0.2163	0.2249
0.7000	0.7630	0.2070	0.2282
0.8000	0.7941	0.1993	0.2306
0.9000	0.8251	0.1928	0.2322
1.0000	0-8507	0.1871	0.2330

Table 2: Numerical solution of the model of CASE 2

T	S(t)	I(t)	R(t)
0	0.5000	0.3226	0.1774
0.1000	0.5323	0.2941	0.1898
0.2000	0.5623	0.2696	0.1997
0.3000	0.5928	0.2487	0.2077
0.4000	0.6210	0.2308	0.2139
0.5000	0.6477	0.2156	0.2186
0.6000	0.6731	0.2026	0.2221
0.7000	0.6970	0.1913	0.2244
0.8000	0.7192	0.1814	0.2258
0.9000	0.7396	0.1724	0.2262
1.0000	0.7582	0.1640	0.2256

Table 3: Numerical solution of the model of CASE 3

T	S(t)	I(t)	R(t)
0	0.5000	0.3226	0.1774
0.1000	0.5300	0.3177	0.1773
0.2000	0.5592	0.3139	0.1772
0.3000	0.5873	0.3113	0.1770
0.4000	0.6146	0.3096	0.1768
0.5000	0.6409	0.3090	0.1766
0.6000	0.6662	0.3093	0.1764
0.7000	0.6907	0.3105	0.1762
0.8000	0.7141	0.3126	0.1760
0.9000	0.7367	0.3154	0.1759
1.0000	0.7584	0.3190	0.1758